Assessing the Importance of Non-Boussinesq Effects in a Coarse Resolution Global Ocean Model.

Martin Losch, Alistair Adcroft, and Jean-Michel Campin

Department of Earth, Atmospheric, and Planetary Sciences, MIT, Cambridge

[Email: mlosch@mit.edu]

1 Overview

- Most conventional general circulation models (GCMs) make the Boussinesq approximation, conserving volume instead of mass.
- Is the Boussinesq approximation justified or even necessary? How do the errors incurred compare with those due to the hydrostatic approximation or errors associated with uncertainties in the physical parameterizations? See, for example, McDougall et al. (2002a).
- We developed a non-Boussinesq GCM by virtue of the isomorphism of the Boussinesq equations in height coordinates and non-Boussinesq equations in pressure coordinates (see Box 2) in the MIT GCM (Marshall et al., 1997a).
- We compare solutions of non-Boussinesq, Boussinesq, and quasi-hydrostatic models after 1000 years of integration (Boxes 3, 4, and 5).

2 Non-Boussinesq Model in Pressure Coordinates and Boussinesq Model in Height Coordinates: Exploiting the Isomorphism in the MITgcm

height coordinates pressure coordinates
dynamical equations

\[ \nabla \cdot (\rho u) = f k \times u + F \]
\[ \frac{\partial u}{\partial t} = -\frac{\nabla \cdot \phi}{\rho} - f k \times u + F \]
\[ -\rho \frac{\partial p}{\partial t} = \frac{\partial}{\partial y} \frac{\partial p}{\partial y} - \frac{\partial}{\partial x} \frac{\partial p}{\partial x} \]
\[ \nabla \cdot u = 0 \]
\[ \frac{\partial u}{\partial t} + \nabla \cdot (u \nabla H) = w \]

with boundary conditions at the surface (\( z = \eta \) and \( p = 0 \))

\[ \frac{\partial u}{\partial t} + \nabla \cdot (u \nabla H) = w \]

with boundary conditions at the bottom (\( z = -H(x, y) \) and \( p = p_0(x, y) \))

\[ \nabla \cdot (\rho u) = f k \times u + F + \frac{\partial}{\partial y} \frac{\partial p}{\partial y} - \frac{\partial}{\partial x} \frac{\partial p}{\partial x} \]

with

\[ \rho \frac{\partial p}{\partial t} + \nabla \cdot (u \rho \nabla p) = \rho \left( w \frac{\partial p}{\partial t} + \nabla \cdot (u \rho \nabla p) \right) \]

\[ \frac{\partial}{\partial y} \frac{\partial p}{\partial y} - \frac{\partial}{\partial x} \frac{\partial p}{\partial x} \]

(De Szoeke and Samelson, 2002, Marshall et al., Climate modeling exploiting atmosphere-ocean fluid isomorphisms, in preparation)

3 Boussinesq Effects on the General Circulation: Sea Surface Height

Figure 1: Top left: Difference of sea surface height variability (square root of the variance over 100 years in centimeters) between the Boussinesq and the non-Boussinesq model. Bottom left: change of sea surface height variability when some non-hydrostatic terms in the horizontal momentum equations and the hydrostatic equation have been included. In the terminology of Marshall et al. (1997b), this is a quasi-hydrostatic model. Above: change in sea surface height variability due to the use of a different implementation of the equation of state; Jackett and McDougall (1995) vs. McDougall et al. (2002b). Clearly, the different equation of state changes the sea surface height variability more than relaxing either the Boussinesq or the hydrostatic approximation.

4 Comparison of Bottom Pressure Variability

Hidrostatic, Boussinesq model in pressure coordinates. Bottom right: difference of bottom pressure variability between the height coordinate model and the pressure coordinate model. Bottom left: difference of bottom pressure variability between the hydrostatic Boussinesq model and a model where some of the non-hydrostatic terms in the horizontal momentum equations and the hydrostatic equation have been included. In the terminology of Marshall et al. (1997b), this is a quasi-hydrostatic model. Bottom right: difference of bottom pressure variability after adding random noise of amplitude \( \frac{22}{10} \times 10^{-6} \) (changing the last digits of a double precision value) to the forcing fields. Clearly, the changes due to the different model formulations are not discernable from the effects of numerical round-off.

5 Relevance to Sea Level Change and Gravity Missions

Figure 3: Mass drift of the height coordinate model and volume drift of the pressure coordinate model, scaled to units of centimeters. The Boussinesq models are volume but not mass conserving and therefore the global mean bottom pressure drifts in time. The non-Boussinesq model in pressure coordinates is mass conserving and recovers a global volume drift caused by steric effects. Clearly, the mass drift of the Boussinesq model can be transformed into a volume drift that is remarkably similar to that of the non-Boussinesq model.

6 Conclusions

- Conventional GCMs make a number of approximations that influence their solution, such as the hydrostatic approximation and the Boussinesq approximations. We find that relaxing the hydrostatic approximation has a larger impact on a coarse resolution global model than do Boussinesq effects.
- Small changes in other approximations, such as the exact form of the equation of state, in physical parameterisations, and numerical noise lead to changes in the circulation, that are at least of the same order of magnitude as those due to Boussinesq effects.
- Because there is no additional cost involved in running a pressure coordinate model, ocean models should be non-Boussinesq. But as far as accuracy is concerned, the Boussinesq approximation is only one of many approximations, and it is certainly not the most severe one.
- Two Caveats:
  - Bottom pressure in pressure coordinates is a prognostic variable, in height coordinates it is diagnostic. Diagnostic variables tend to exhibit greater variability, thus biasing the results.
  - Details of the comparison are incomplete. For example, the vertical viscosity and diffusivity in both models are slightly different for technical reasons. This may be the largest contribution to the current differences between the Boussinesq and non-Boussinesq model.

References


Dougall, D. and Dougall, K. (1999). Non-Boussinesq approximations. We find that relaxing the hydrostatic approximation has a larger impact on a coarse resolution global model than do Boussinesq effects.

Dougall, D. and Dougall, K. (1999). Non-Boussinesq approximations. We find that relaxing the hydrostatic approximation has a larger impact on a coarse resolution global model than do Boussinesq effects.

Dougall, D. and Dougall, K. (1999). Non-Boussinesq approximations. We find that relaxing the hydrostatic approximation has a larger impact on a coarse resolution global model than do Boussinesq effects.

Dougall, D. and Dougall, K. (1999). Non-Boussinesq approximations. We find that relaxing the hydrostatic approximation has a larger impact on a coarse resolution global model than do Boussinesq effects.

Dougall, D. and Dougall, K. (1999). Non-Boussinesq approximations. We find that relaxing the hydrostatic approximation has a larger impact on a coarse resolution global model than do Boussinesq effects.

Dougall, D. and Dougall, K. (1999). Non-Boussinesq approximations. We find that relaxing the hydrostatic approximation has a larger impact on a coarse resolution global model than do Boussinesq effects.

Dougall, D. and Dougall, K. (1999). Non-Boussinesq approximations. We find that relaxing the hydrostatic approximation has a larger impact on a coarse resolution global model than do Boussinesq effects.

Dougall, D. and Dougall, K. (1999). Non-Boussinesq approximations. We find that relaxing the hydrostatic approximation has a larger impact on a coarse resolution global model than do Boussinesq effects.

Dougall, D. and Dougall, K. (1999). Non-Boussinesq approximations. We find that relaxing the hydrostatic approximation has a larger impact on a coarse resolution global model than do Boussinesq effects.

Dougall, D. and Dougall, K. (1999). Non-Boussinesq approximations. We find that relaxing the hydrostatic approximation has a larger impact on a coarse resolution global model than do Boussinesq effects.